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Geometric Construction of 4-Finger Force-Closure Grasps for Polyhedral Objects

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1. Introduction

A force-closure grasp has the property to reject external forces applied on the grasped object by means of the forces applied by the fingers. The theory regarding force-closure grasps has been deeply studied, and different techniques have been proposed for different cases [1-10].

The force-closure grasps with four non-planar contact forces are classified into three categories [8]: *concurrent*, *flat-pencil* and *regulus*. In a *concurrent* grasp the lines of action of the four contact forces intersect in a point (Figure 1a). In a *flat-pencil* grasp the lines of action of two contact forces intersect in a point and those of the other two forces intersect in another point, with these two points laying on the intersection of the planes defined by each pair of lines of action (Figure 1b). In a *regulus* grasp two lines of action are on different sides of a plane parallel to them and at a distance *d* to this plane, in the same way the other two lines of action are on different sides of a plane parallel to them and at a distance *d* to this plane, and the projections of each pair of lines of actions on their corresponding parallel plane must form a *concurrent* or a *flat-pencil* (*regulus* grasp, Figure 1c).

Ponce et al. [8] developed a technique to build *concurrent* grasps, they determine the sets of four faces whose relative orientations satisfy a sufficient condition and their relative positions allow this type of grasp, and then the set of faces that optimizes an objective function is selected to be contacted by the fingers. However, the method does not work for *flat-pencil* and *regulus* grasps.

Sundang and Ponce [9] proposed a method for the construction of the three types of non-planar grasps over four faces whose relative orientations satisfy a sufficient condition but assuming that the relative positions of the faces to be contacted by the fingers allow *flat-pencil* and *regulus* grasps.

Yoshikawa [12] developed a model to determine the internal forces for the three types of non-planar grasps considering that the contact points are known.

In this work, a method to build the three types of non-planar grasps is proposed. First, the sets of four object faces whose relative orientations satisfy a necessary and sufficient condition and whose relative positions allow the existence of three (*concurrent*, *flat-pencil* and *regulus*) or two (*flat-pencil* and *regulus*) types of non-coplanar grasps are selected (the determination of sets of four faces that only allow *regulus* grasp is not considered in this paper). Second, from these sets the one that maximizes a quality function is selected and, finally, on the selected faces four contact points assuring a force-closure grasp are determined according to the possible types of non-planar grasps.

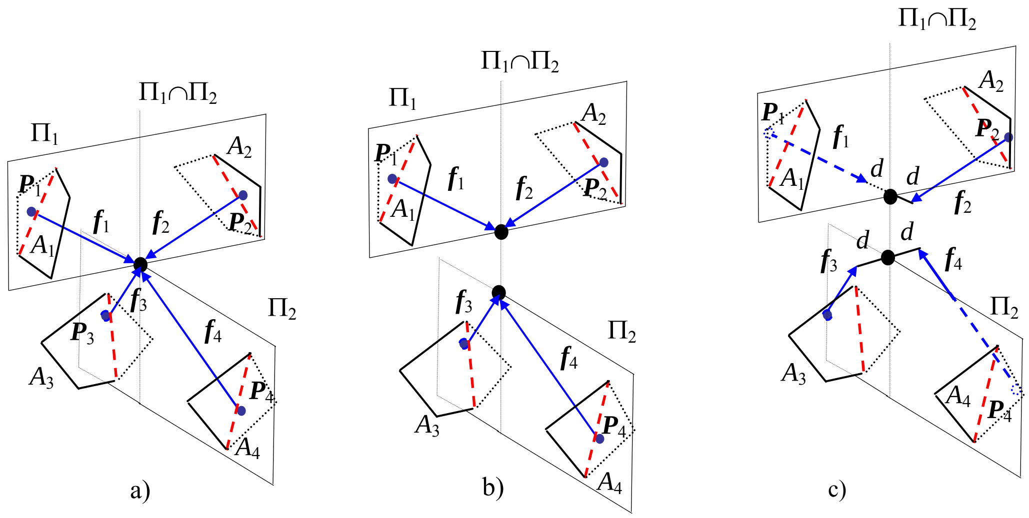

Fig. 1. Types of grasps: a) *Concurrent*; b) *flat-pencil*; c) *regulus*.

2. Assumptions and basic nomenclature

The following assumptions are considered in this work:

- The objects are polyhedrons.
- The grasp is done using four fingers and each finger contacts with a different face of the object.
- Only the fingertips will contact with the object surface and the contact is a point (then for stability reasons, the contact points cannot be on an object edge).
- The friction coefficient μ is constant.

The following basic nomenclature will be used:

P_i: contact point on the object surface (i =1,2,3,4).

A_i: contacted face of the object (i =1,2,3,4).

$\boldsymbol{n}_i$: unitary vector with object inward direction normal to A_i.

$\alpha=\mathrm{tg}^{-1}\mu$: half-angle of the friction cone ($\alpha<\pi/2$).

C_{fi}: friction cone with half-angle α, axis parallel to $\boldsymbol{n}_i$ and vertex at P_i.

C_i: friction cone with half-angle α, axis parallel to $\boldsymbol{n}_i$ and vertex at the origin of the reference system (representation of C_{fi} in the force space).

$\boldsymbol{f}_i$: contact force applied at contact P_i ($\boldsymbol{f}_i\subset C_{fi}$).

c_m: object center of mass.

3. Force-closure grasp

A force-closure grasp (FCG) must satisfy [3]:

$$\sum_{i=1}^{n} \boldsymbol{f}_i = \boldsymbol{F}_{ex} \qquad \text{and} \qquad \sum_{i=1}^{n} \boldsymbol{r}_i \times \boldsymbol{f}_i = \boldsymbol{M}_{ex} \tag{1}$$

where n is the number of contact points, $\boldsymbol{r}_i$ is the vector from the object center of mass to the contact point P_i, and $\boldsymbol{F}_{ex}$ and $\boldsymbol{M}_{ex}$ are, respectively, any external arbitrary force and torque applied on the object.

A necessary condition for the existence of a FCG is that equations (1) must be satisfied for $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0 [3]-[8]. In the case of a FCG with four $\boldsymbol{f}_i$ without any three of them acting in the same plane, the following two conditions must be satisfied when $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0 [8][9][10].

C1: $\boldsymbol{f}_1$, $\boldsymbol{f}_2$, $\boldsymbol{f}_3$ and $\boldsymbol{f}_4$ span $\Re^3$.

C2: The lines of action of the applied forces form a *concurrent, flat-pencil*, or *regulus* grasp.

Ponce et al. (1997) demonstrated that if $\boldsymbol{f}_i \in C_{fi}$, i=1,2,3,4, satisfy the conditions *C1* and *C2*, then the contact points allow a FCG. Also, if a set of four faces allows a *concurrent* grasp with the contact points in the interior of the face (i.e. the contacts do not belong to the face boundary) then it is always possible to determine *flat-pencil* and *regulus* grasps on the same set of faces; moreover, the different types of grasp can be reached using the same directions of force by changing only the contact points. In the same way, if a set of four faces allows a *flat-pencil* grasp (but not necessarily a *concurrent* grasp) then it is always possible to determine a *regulus* grasp on the same set of faces.
The approach presented in this report determines non-coplanar grasps on sets of faces that allow at least two types of non-coplanar grasps, i.e. the sets of four faces that allow only *regulus* grasp are not considered in this work.

4. Selection of the sets of four faces that allow FCG

The selection of the sets of four object faces that allow at least two types of non-planar grasps is done in two phases:

1. Selection of faces according to their orientations.
2. Selection of faces according to their positions (from those passing the first phase).

Each phase is described in the following subsections.

4.1 Selection of faces according to their orientations

In this phase the sets of four faces whose relative orientations allow the application of forces $\boldsymbol{f}_i \in C_i$ i=1,2,3,4 that span $\Re^3$ are selected. Then, for each of these sets of faces, subsets ${}^*C$ of the friction cones C_i are determined such that, if $\boldsymbol{f}_i \in {}^*C_i$ i=1,2,3,4, the four $\boldsymbol{f}_i$ span $\Re^3$ with independence of the contact point. A sufficient condition for the existence of $\boldsymbol{f}_i \in C_i$, i=1,2,3,4, that span $\Re^3$ is that $\boldsymbol{0} \in ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$ [8]. Therefore the sets of faces that satisfy this condition are selected as candidates for a FCG. However, due to friction, there exist sets of faces with $\boldsymbol{0} \notin ConvexHull(\boldsymbol{n}_i)$ that also allow applied forces $\boldsymbol{f}_i \in C_i$, i=1,2,3,4, that span $\Re^3$. These sets of faces satisfy the conditions in Proposition 1 below and are also considered as candidates for a FCG.
Let:

Π_l be the closest plane to the origin that contains a face of $ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$, i=1,2,3,4 (Figure 2).

φ be the angle between Π_l and any of the three $\boldsymbol{n}_i$ that determines Π_l (note that φ is the same for any i).

$'\Pi_l$ be the plane parallel to Π_l that contains the origin.

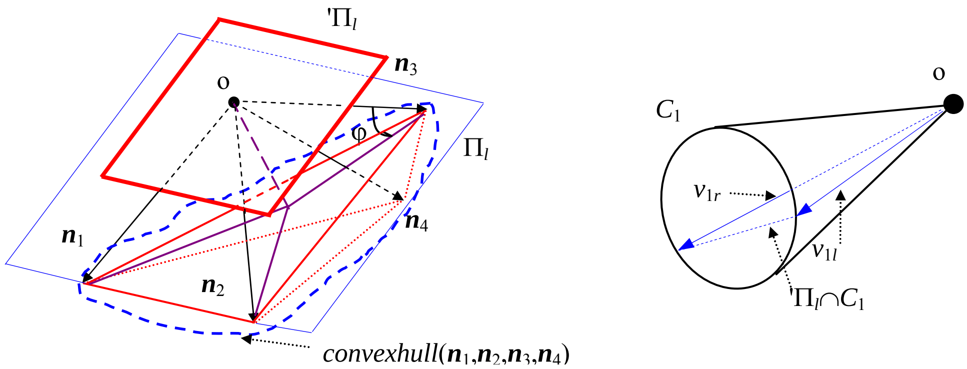

Fig. 2. Selection of faces according to their orientations for a set of faces with $\mathbf{0} \notin ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$.

$\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ be the two unitary vectors that indicate the two boundary directions of $'\Pi_l \cap C_i$, i=1,2,3,4, respectively ($\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ are not defined when $'\Pi_l \cap C_i = \varnothing$ and $\boldsymbol{v}_{il} = \boldsymbol{v}_{ir}$ when C_i is tangent to $'\Pi_l$).

***Proposition* 1**. Given four faces such that $\mathbf{0} \notin ConvexHull(\boldsymbol{n}_1,\boldsymbol{n}_2,\boldsymbol{n}_3,\boldsymbol{n}_4)$, $\exists\, \boldsymbol{f}_i \in C_i$, i=1,2,3,4 spanning $\Re^3$ iff:

1. $\varphi < \alpha$.
2. $\mathbf{0} \in ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r}, \boldsymbol{v}_{4l}, \boldsymbol{v}_{4r})$ ■

Proof. If $\varphi \geq \alpha$ then all four cones C_i lie in one of the half-spaces of $\Re^3$ defined by $'\Pi_l$ and therefore the vectors in the other half-space can not be obtained as a linear combination of any four vectors from the cones C_i (note that $\boldsymbol{v}_{il}$ and $\boldsymbol{v}_{ir}$ do not exist for $\varphi > \alpha$). Then $\varphi < \alpha$ must be satisfied.

If $\varphi < \alpha$ and $\mathbf{0} \notin ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r}, \boldsymbol{v}_{4l}, \boldsymbol{v}_{4r})$ then the plane $'\Pi_l$ can not be spanned by a linear combination of the components on $'\Pi_l$ of any four vectors from the cones C_i, and therefore some vectors of $\Re^3$ cannot be obtained.

If $\varphi < \alpha$ and $\mathbf{0} \in ConvexHull(\boldsymbol{v}_{1l}, \boldsymbol{v}_{1r}, \boldsymbol{v}_{2l}, \boldsymbol{v}_{2r}, \boldsymbol{v}_{3l}, \boldsymbol{v}_{3r}, \boldsymbol{v}_{4l}, \boldsymbol{v}_{4r})$ then the plane $'\Pi_l$ can be spanned by a linear combination of the components on $'\Pi_l$ of four vectors from the friction cones C_i and, at the same time, there are force components in the two half-spaces of $\Re^3$ defined by $'\Pi_l$, as a consequence any vector of $\Re^3$ can be obtained as a linear combination of four vectors, one from each C_i. ■

After selecting the set of faces that allow FCG, the subsets $^{*}C_i$ of the friction cones directions C_i that assure the FCG have to be determined. It is done as follows. Let:

^{+}S and ^{-}S be the two half-spaces defined by $'\Pi_l$ (Figure 3a), with ^{+}S containing the $\boldsymbol{n}_i$ that does not define Π_l.

$^{+}C_i$ and $^{-}C_i$ be the two largest cones contained in $C_i \cap {}^{+}S$ and $C_i \cap {}^{-}S$, respectively (if $C_i \cap {}^{+}S = \varnothing$ then $^{+}C_i$ does not exist and $^{-}C_i = C_i$, and vice versa if $C_i \cap {}^{-}S = \varnothing$).

$^{+}\boldsymbol{n}_i$ and $^{-}\boldsymbol{n}_i$ be unitary vectors along the axis of $^{+}C_i$ and $^{-}C_i$ respectively (if $^{+}C_i = C_i \Rightarrow {}^{+}\boldsymbol{n}_i = \boldsymbol{n}_i$ and if $^{-}C_i = C_i \Rightarrow {}^{-}\boldsymbol{n}_i = \boldsymbol{n}_i$).

Now, four unitary non-coplanar vector $^{*}\boldsymbol{n}_i \in C_i$, i=1,2,3,4, are computed as:

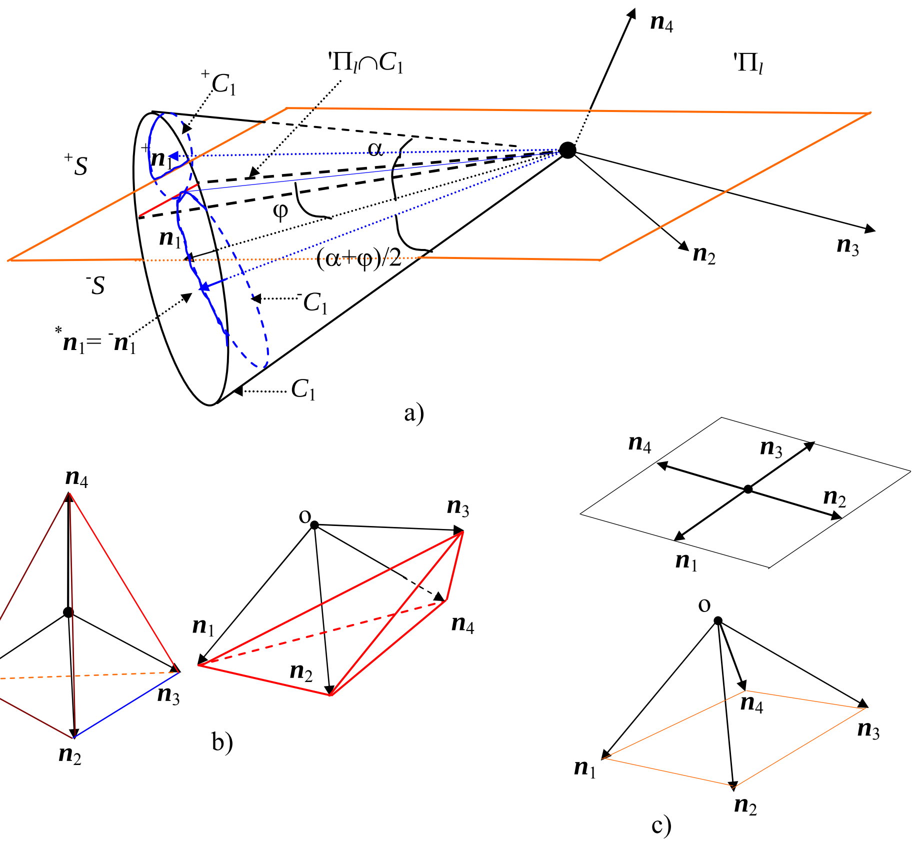

Fig. 3. a) Determination of $^*\boldsymbol{n}_1$; b) two cases where the extremes of $\boldsymbol{n}_i$ are not coplanar; c) two cases where the extremes of $\boldsymbol{n}_i$ are coplanar.

- If the extremes of $\boldsymbol{n}_i$ i=1,2,3,4 are non-coplanar (Figure 3b), then three $^*\boldsymbol{n}_i$ are equal to the $^-\boldsymbol{n}_i$ corresponding to the three $\boldsymbol{n}_i$ that define Π_l, and the fourth $^*\boldsymbol{n}_i$ is equal to the $^+\boldsymbol{n}_i$ of the remaining $\boldsymbol{n}_i$ (i.e. the one that does not defines Π_l).
- If the extremes of $\boldsymbol{n}_i$ i=1,2,3,4 are coplanar (Figure 3c), then the convex hulls defined by the sets of four vectors $^-\boldsymbol{n}_i$ or $^+\boldsymbol{n}_i$ i=1,2,3,4 are computed. Then, $^*\boldsymbol{n}_i$ i=1,2,3,4 are equal to the corresponding four vectors (either $^-\boldsymbol{n}_i$ or $^+\boldsymbol{n}_i$) that determine the convex hull with largest volume.

Let now:

Π_{jkr} be the plane parallel to the triangle defined by the extremes of $^*\boldsymbol{n}_j$, $^*\boldsymbol{n}_k$ and $^*\boldsymbol{n}_r$ for $j,k,r\in\{1,2,3,4\}$ with $j\neq k\neq r$, and passing through the origin (since the extremes of $^*\boldsymbol{n}_i$ i=1,2,3,4, are not coplanar there exist four different Π_{jkr}, Figure 4a).

^+S_i and ^-S_i be the two half-spaces defined by Π_{jkr}, where $^*\boldsymbol{n}_i\in{}^+S_i$, $i,j,k,r\in\{1,2,3,4\}$ with $i\neq j\neq k\neq r$.

The independent subset $^*C_i$ of each friction cone C_i is determined such that $^*C_i\subset{}^+S_i$ and $^*C_j$, $^*C_k$ and $^*C_r$ are included in ^-S_i, therefore $^*C_i\subset{}^+S_i\cap{}^-S_j\cap{}^-S_k\cap{}^-S_r$.

$^+S_i\cap{}^-S_j\cap{}^-S_k\cap{}^-S_r$ determines a polyhedral convex cone T_i of three faces, each one on a plane Π_{ijk}, i=1,…,4 and $j,k\in\{1,2,3,4\}$ with $i\neq j\neq k$. For instance, the faces of T_4 (Figure 4a) are on the planes Π_{124}, Π_{134} and Π_{234}, respectively, and their edges are on the intersections of each pair of these planes. Π_{123} doest not determine T_4, but determines T_1, T_2 and T_3, respectively and defines the half-spaces $^+S_4$ and $^-S_4$ such that $T_4\subset{}^+S_4$ and T_1, T_2 and T_3 are included in $^-S_4$.

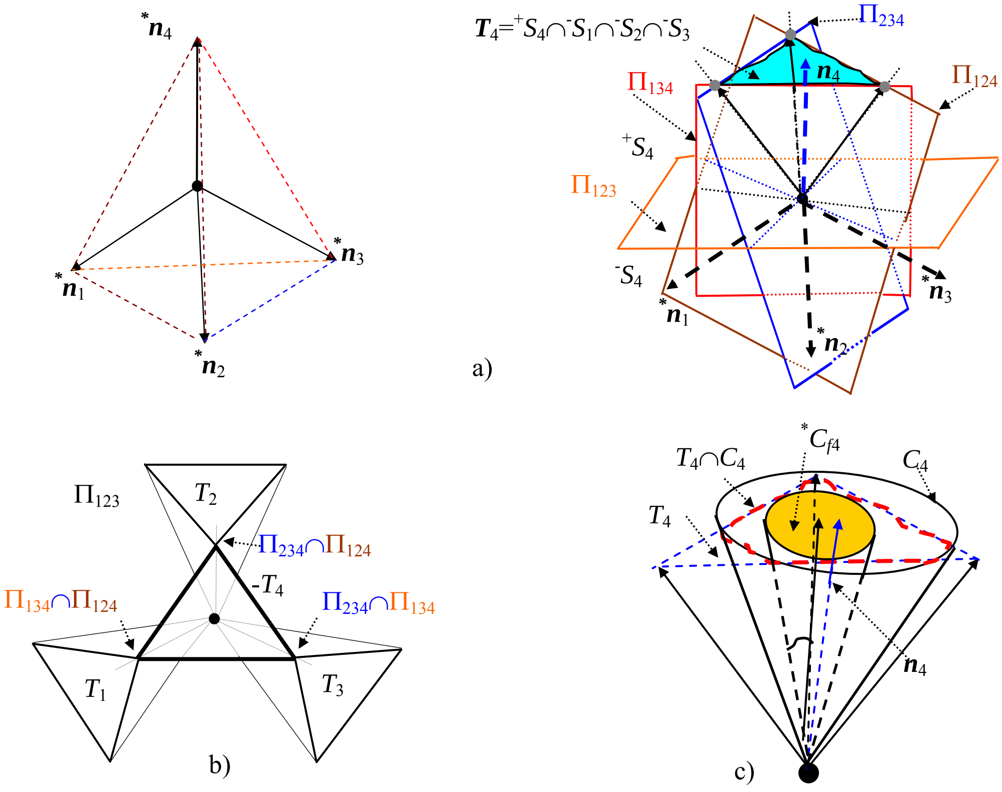

Fig. 4. Determination of: a) the plane Π_{jkr}; b) cone T_4; c) $^*C_{f4}$.

Since the four convex cones T_i (i=1,…,4) are determined by the four planes Π_{jkr} ($j,k,r\in\{1,2,3,4\}$ with $j\neq k\neq r$), then it is always satisfied that the three edges of the negated of T_i, represented as $-T_i$ (Figure 4b), are respectively an edge of T_j, T_k and T_r. This implies that any vector that belongs to $-T_i$ can be obtained as a linear combination of three vectors, one from T_j, T_k and T_r, respectively. As a consequence any four vectors, one from each T_i, i=1,2,3,4, always span $\Re^3$. Note that by construction $T_i\neq\varnothing$, i=1,2,3,4, even if $\mathbf{0}\notin ConvexHull(^*\boldsymbol{n}_1, {}^*\boldsymbol{n}_2, {}^*\boldsymbol{n}_3, {}^*\boldsymbol{n}_4)$.

The conditions satisfied by the orientations of the faces to allow a FCG assures that $T_i\cap C_i\neq\varnothing$, then $^*C_i$ is determined as $^*C_i=T_i\cap C_i$ and it is non null. Finally, each set of directions $^*C_i$ is approximated by the largest cone included in $T_i\cap C_i$ (Figure 4c).

4.2 Selection of faces according to their positions

From previous section, it must be satisfied $\boldsymbol{f}_i\in{}^*C_i\subset C_i$, i=1,2,3,4, in order to allow the FCG with independence of the particular contact points on the object faces. In order to have the largest range of variation of the directions of $\boldsymbol{f}_i$ to keep a FCG when $\boldsymbol{F}_{ex}\neq 0$ and $\boldsymbol{M}_{ex}\neq 0$, it is desirable the direction of $\boldsymbol{f}_i$ to be aligned with the axis, $\boldsymbol{n}_{fi}$, of $^*C_i$ when $\boldsymbol{F}_{ex}=0$ and $\boldsymbol{M}_{ex}=0$, and so is considered as a constraint in this selection of faces.

Now, given a set of four faces A_i, i=1,2,3,4, the procedure to test if it is valid to produce a FCG is the following:

1. Determine the volumen, D_i, swept by A_i when A_i is displaced in the direction of $\boldsymbol{n}_{fi}$, i=1,2,3,4, (Figure 5).
2. Compute $D_j\cap D_k$, for any $j\in\{1,2,3,4\}$ and $\forall k\in\{1,2,3,4\}$ with $j\neq k$. If $\forall k$ $D_j\cap D_k=\varnothing$ then return (***Invalid***).

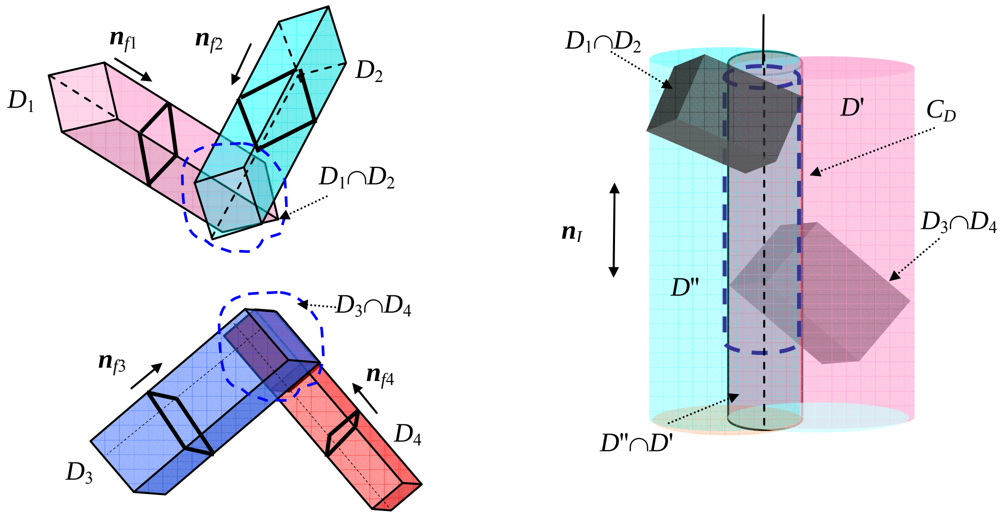

Fig. 5. Selection of faces according their relative positions.

In this case it is not possible for the line of action of $\boldsymbol{f}_j$, with direction of $\boldsymbol{n}_{fj}$, to intersect with the line of action of any $\boldsymbol{f}_k$, with the direction of $\boldsymbol{n}_{fk}$.

3. Determine two pairs $(D_j \cap D_k)$ and $(D_r \cap D_h)$, where at least $D_j \cap D_k \neq \varnothing$, with $\{j,k,r,h\}=\{1,2,3,4\}$. Consider the plane define by $\boldsymbol{n}_{fj}$ and $\boldsymbol{n}_{fk}$ and the plane defined by $\boldsymbol{n}_{fr}$ and $\boldsymbol{n}_{fh}$, and let $\boldsymbol{n}_I$ be the vector parallel to intersection of these two planes.
4. Determine the volumes, D' and D'', swept by $D_j \cap D_k$ and $D_r \cap D_h$ with $\{j,k,r,h\}=\{1,2,3,4\}$, respectively, when they are displaced in the direction of $\boldsymbol{n}_I$.
   - If $D' \cap D'' = \varnothing$ then return (***Invalid***).

     In this case, the intersection point of the lines of action of $\boldsymbol{f}_j$ and $\boldsymbol{f}_k$ and the intersection point of the lines of action of $\boldsymbol{f}_r$ and $\boldsymbol{f}_h$ do not lay on the intersection of the planes defined by each pair of lines of action (note that this intersection is always parallel to $\boldsymbol{n}_I$).
   - If $D' \cap D'' \neq \varnothing$ then return (***Valid***).

     In this case, the projections on A_i with directions of $\boldsymbol{n}_{fi}$ of any two points contained in a straight line parallel to $\boldsymbol{n}_I$ and belonging to $D' \cap D'' \cap D_j \cap D_k$ and $D' \cap D'' \cap D_r \cap D_h$ respectively, always determine a FCG.

If a valid set of faces satisfies $D_1 \cap D_2 \cap D_3 \cap D_4 \neq \varnothing$ (note that $D_1 \cap D_2 \cap D_3 \cap D_4 \subset D' \cap D''$) then it allows *concurrent*, *flat pencil* and *regulus* grasps; otherwise it only allows *flat pencil* and *regulus* grasps.

This is a conservative approach because it selects sets of four faces that allow FCG and also must allow to apply $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$ that allow at least two types of non-coplanar grasps, therefore there may be sets of faces that actually allow FCG that are not considered as valid in this procedure.

5. Quality of the sets of faces

In order to select the set of faces to be contacted by the fingers, all valid sets of faces are evaluated according to a quality measure that considers:

- The tetrahedron defined by P_1, P_2, P_3 and P_4 should have the maximum possible volume and its centroid should be as close as possible to the object center of mass. This produces better results in front of gravitational forces and torques [4][8].
- The forces $\boldsymbol{f}_i$ should have similar modules in absence of external perturbations (i.e. for $\boldsymbol{F}_{ex}$=0 and $\boldsymbol{M}_{ex}$=0). This produces a larger range of variation of the applied forces to keep the FCG when external perturbations exist [11].

Consider C_D=*convexhull*($D'\cap D''\cap D_j\cap D_k$, $D'\cap D''\cap D_r\cap D_h$) (all the pairs of points whose projections on A_i, with direction of $\boldsymbol{n}_{fi}$, determine a FCG lie in C_D). The quality function uses the centroid, c_d, and the volume, V_c, of C_D, as well as the distance, d_c, from c_d to the object center of mass c_m.

The quality function that returns the quality of a set of faces as a value in the range [0,1] (being 1 the highest quality) is

$$Q = \prod_{i=1}^{3} q_i \tag{2}$$

with:

$$q_1 = \left| \frac{d_{c\max} - d_c}{d_{c\max}} \right| \tag{3}$$

where d_{cmax} is the maximum value of d_c from all the valid sets of faces (q_1 indicates how close is c_m from c_d);

$$q_2 = \left| \frac{V_c}{V_{c\max}} \right| \tag{4}$$

where V_{cmax} is the maximum value of V_c from all the valid sets of faces which allow a FCG (q_2 indicate how close is V_c from V_{cmax});

$$q_3 = \left| \frac{V_e}{V_{e\max}} \right| \tag{5}$$

where V_e is the radius of the largest sphere centered at the origin and included in *convexhull*($\boldsymbol{n}_{f1}$,$\boldsymbol{n}_{f2}$,$\boldsymbol{n}_{f3}$,$\boldsymbol{n}_{f4}$), and V_{emax}=0.5 is the maximum possible value of V_e, and it is obtained in the particular case where the angle between any two $\boldsymbol{n}_{fi}$ is 120º. If q_3=1 then the forces $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$, i=1,2,3,4, have the same modules.

The set of faces with the largest Q is selected for the grasp.

6. Determination of the contact points

The positions of P_i, i=1,2,3,4, on the selected faces are determined such that the centroid of the tetrahedron that they define is close to the object center of mass c_m. The procedure for the determination of the contact points depends on the type of non-coplanar grasp as follows. Let c_t be the centroid of $D_T=D_1\cap D_2\cap D_3\cap D_4$.

Case of concurrent grasps (Figure 6a):

P_i, i=1,2,3,4, is determined such that the intersection point of the lines actions of $\boldsymbol{f}_i$ with direction of $\boldsymbol{n}_{fi}$, is closest to c_m.

1. Determine the point $P_I \in D_T\cap \overline{c_t c_m}$ closest to c_m.

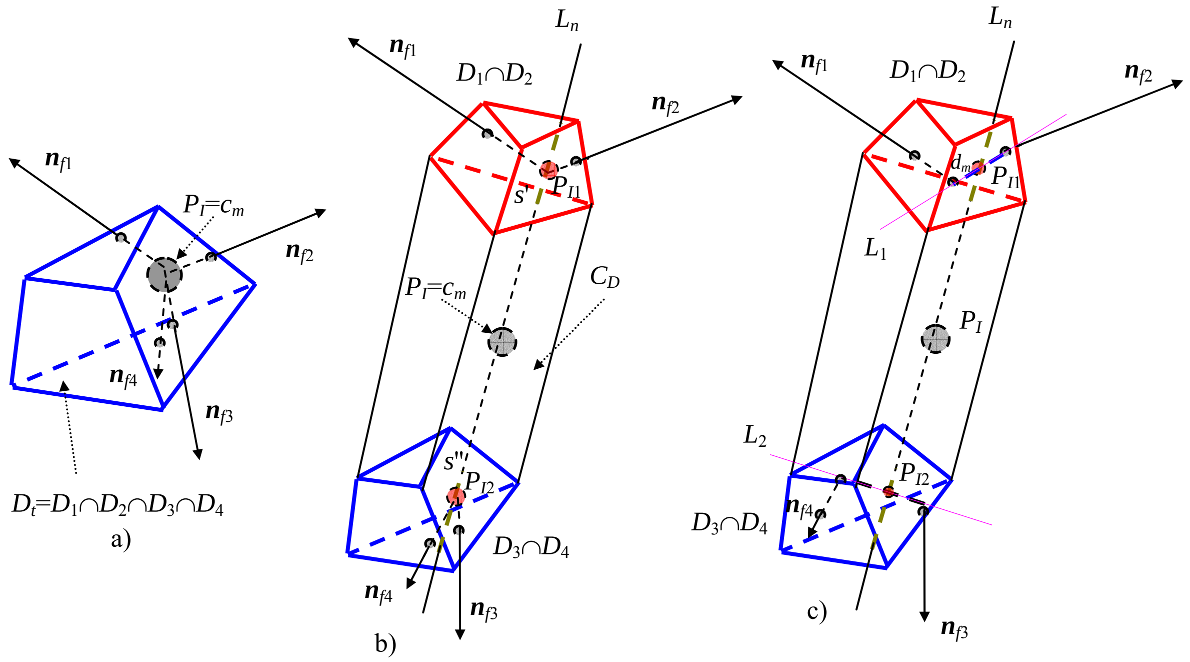

Fig. 6. Determination of a grasp: a) *concurrent*; b) *flat-pencil*; c) *regulus*

2. Trace four straight lines through P_I with the directions of $\boldsymbol{n}_{f1}$, $\boldsymbol{n}_{f2}$, $\boldsymbol{n}_{f3}$ and $\boldsymbol{n}_{f4}$. The intersection points of these straight lines with A_1, A_2, A_3, and A_4 determine P_1, P_2, P_3 and P_4, respectively.

Case of flat-pencil grasps (Figure 6b):

P_i, i=1,2,3,4, is determine such that the straight line that contains the two intersection points of the lines de action of $\{\boldsymbol{f}_j, \boldsymbol{f}_k\}$ and of $\{\boldsymbol{f}_r, \boldsymbol{f}_h\}$ respectively, is closest to c_m.

1. Compute the point $P_I \in C_D \cap \overline{c_d c_m}$ closest to c_m.

2. Trace a straight line, L_n, through P_I with direction of $\boldsymbol{n}_I$. Since $P_I \in C_D$ and $L_n // \boldsymbol{n}_I$ then $s' = L_n \cap D_j \cap D_k \neq \emptyset$ and $s'' = L_n \cap D_r \cap D_h \neq \emptyset$, with $\{j,k,r,h\}=\{1,2,3,4\}$.

   Let P_{I1} and P_{I2} be the midpoints of s' and s'', respectively.

3. Trace two straight lines through P_{I1} with directions of $\boldsymbol{n}_{fj}$ and $\boldsymbol{n}_{fk}$ respectively, and another two through P_{I2} with directions of $\boldsymbol{n}_{fr}$ and $\boldsymbol{n}_{fh}$, $\{j,k,r,h\}=\{1,2,3,4\}$, respectively. The intersection points of these four straight lines with the corresponding faces A_1, A_2, A_3, and A_4 determine P_1, P_2, P_3 and P_4, respectively.

Case of regulus grasps (Figure 7):

P_i, i=1,2,3,4, is determine such that the intersection of the two planes parallels and equidistant to the lines de action of $\{\boldsymbol{f}_j, \boldsymbol{f}_k\}$ and of $\{\boldsymbol{f}_r, \boldsymbol{f}_h\}$ respectively, is closest to c_m.

1. Repeat the steps 1 to 2 described above for the case of *flat-pencil* grasps.

2. Trace a straight line, L_1, through P_{I1} with direction of $\boldsymbol{n}_{fj} \times \boldsymbol{n}_{fk}$ and another straight line, L_2, through P_{I2} with direction of $\boldsymbol{n}_{fr} \times \boldsymbol{n}_{fh}$. Since $P_{I1} \in D_j \cap D_k$ and $P_{I2} \in D_r \cap D_h$ then $s_1 = L_1 \cap D_j \cap D_k \neq \emptyset$ and $s_2 = L_2 \cap D_r \cap D_h \neq \emptyset$, with $\{j,k,r,h\}=\{1,2,3,4\}$.

3. Compute the minimum distance, d_{mx}, from P_{I1} to the extremes of s_x. Of the same way compute the minimum distance, d_{my}, from P_{I2} to the extremes of s_y.

   Let d_m be the minimum from (d_{mx}, d_{my}).

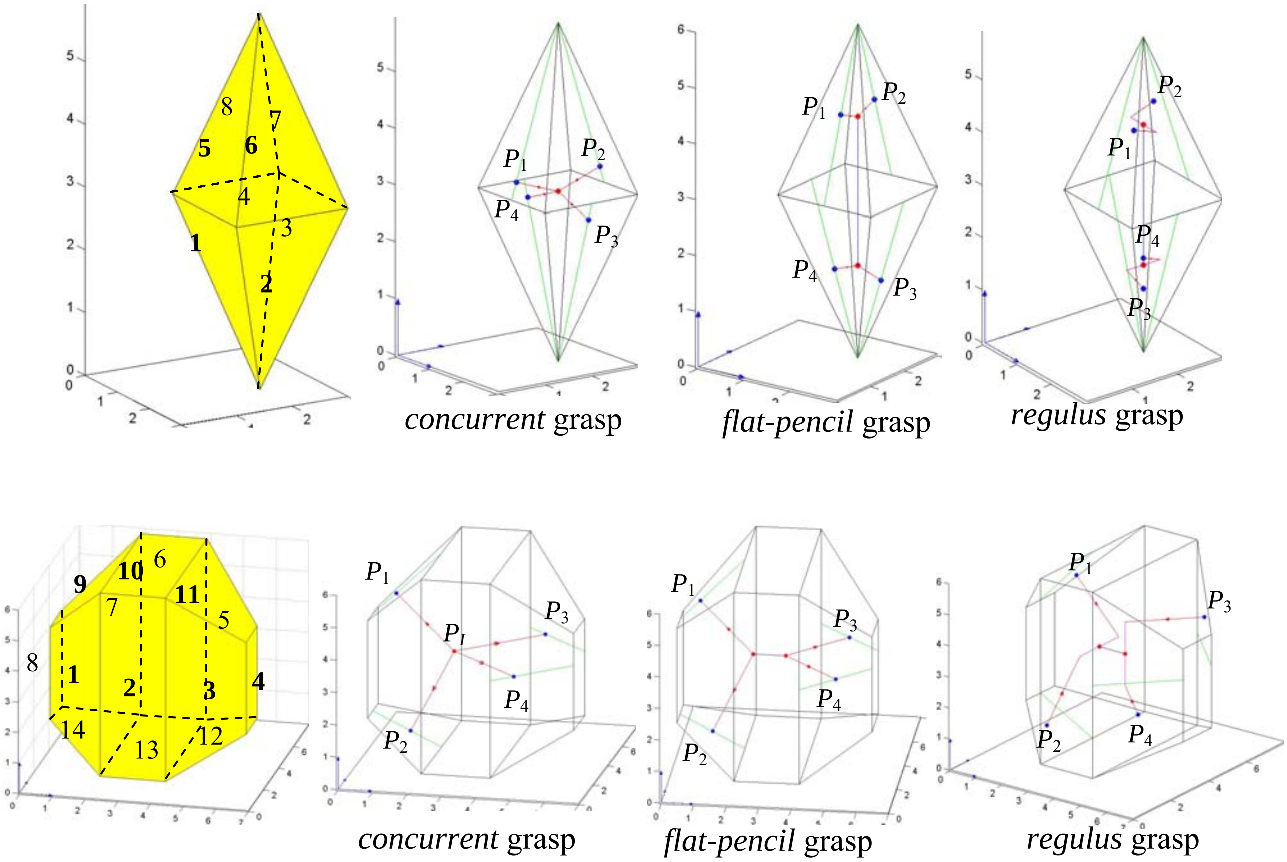

Fig. 8. Two examples of FCG, obtained with the proposed approach, where the selected set of faces allows the three types of non-coplanar grasps.

4. Trace a segment parallel to $\boldsymbol{n}_{f1}\times\boldsymbol{n}_{f2}$, of longitude $2d_m$ such that P_{I1} is their midpoint, and another segment parallel to $\boldsymbol{n}_{f3}\times\boldsymbol{n}_{f4}$, of longitude $2d_m$ and P_{I2} is their midpoint (P_{I1} and P_{I2} lay in L_n).
5. Trace a straight line through each extreme of the two segments determined in the previous step with the direction of $\boldsymbol{n}_{f1}$, $\boldsymbol{n}_{f2}$, $\boldsymbol{n}_{f3}$ and $\boldsymbol{n}_{f4}$. The intersection points of these straight lines with A_1, A_2, A_3, and A_4 determine P_1, P_2, P_3 and P_4, respectively.

P_i, i=1,2,3,4, as they were obtained for the three types of non-coplanar grasps, allow to apply $\boldsymbol{f}_i\in C_{fi}$ with direction of $\boldsymbol{n}_{fi}$ to reach the equilibrium in absence of external perturbations; nevertheless, a positive linear combination of these forces may not necessarily satisfy equation (1) for any $\boldsymbol{F}_{ex}\neq\emptyset$ and $\boldsymbol{M}_{ex}\neq\emptyset$, but due to friction it can be assured that P_i allow to apply the necessary $\boldsymbol{f}_i\in C_{fi}$ (possibly with directions different from $\boldsymbol{n}_{fi}$) to satisfy equation (1) for any $\boldsymbol{F}_{ex}\neq\emptyset$ and $\boldsymbol{M}_{ex}\neq\emptyset$ [8]-[9].

7. Examples

Four examples are shown to illustrate the proposed approach. In all the cases it is assume a constant friction coefficient μ=0,25. The implementation was done using Matlab and executed on a server INTEL Biprocessor Pentium III 1,4 GHz. The Figure 8 shows two objects where the selected set of faces allows the three types of non-planar grasp. The Figure 9 shows two objects where the selected set of faces only allows *flat-pencil* and *regulus* grasps. The Figure 10 shows the three types of non-coplanar FCG obtained on a set of four faces with coplanar normal directions.

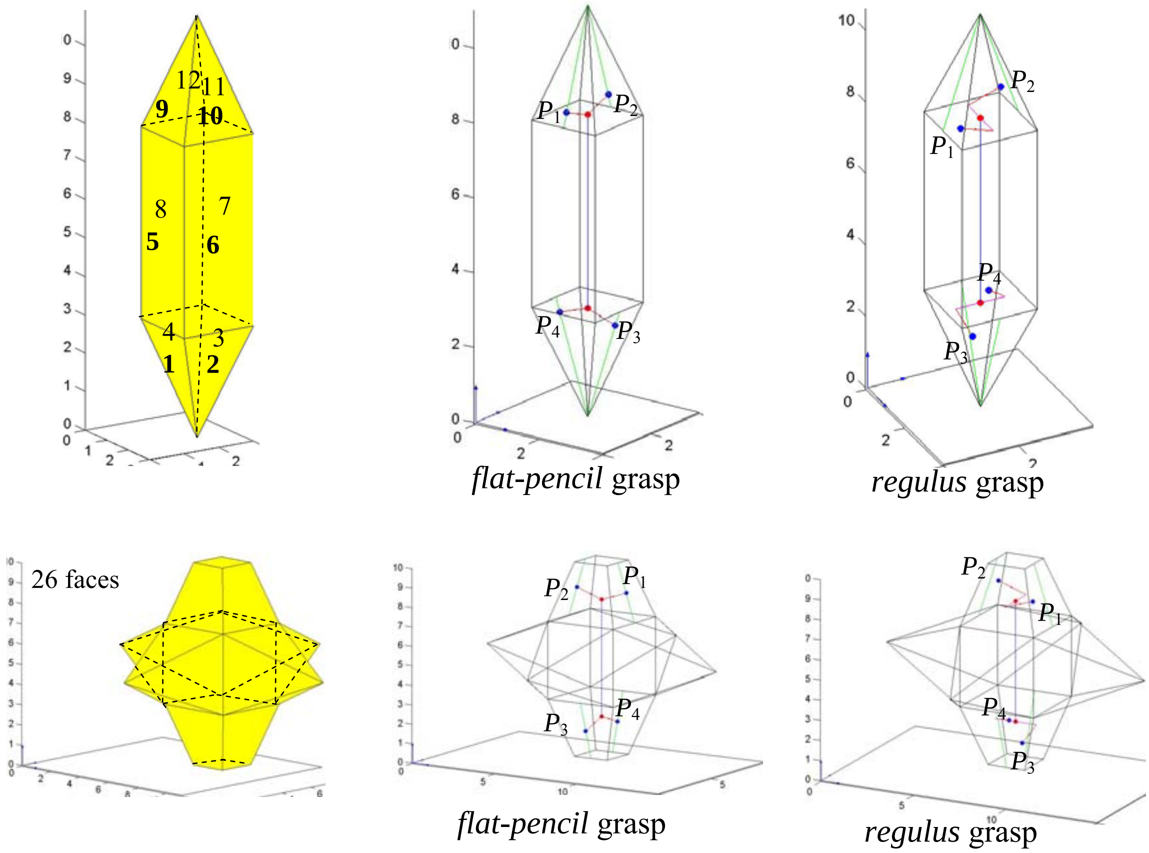

Fig.9. Two examples of FCG, obtained by the proposed approach, the selected set of faces in each object, only allows *flat-pencil* and *regulus* grasps.

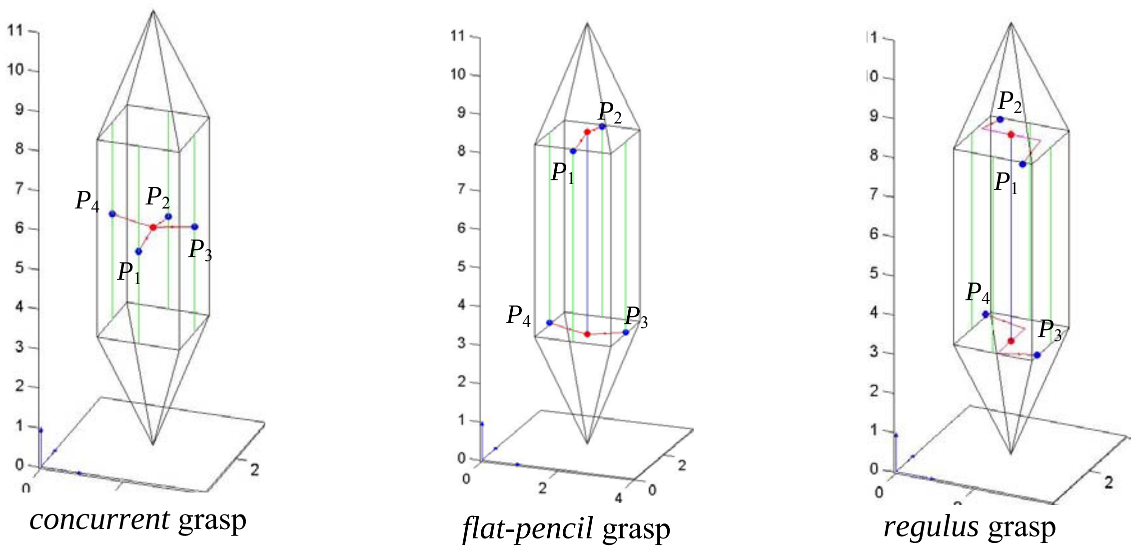

Fig. 10. Determination of FCG on a set of faces with coplanar $\boldsymbol{n}_i$

8. Conclusion

The geometric method presented in this paper allows the construction of the three types of non-planar grasp: *concurrent*, *flat-pencil* and *regulus*. The sets of four faces whose relative orientations satisfy a necessary and sufficient condition and their relative positions allow the contact forces applied by the fingers to produce at least two type of non-planar grasp are considered as valid for a FCG. Among these sets of faces, the one that maximizes a quality function is selected. Then, on the faces of the selected set, according to the possible types of non-planar grasp, four contact points assuring a force-closure grasp are determined. Since all the possible sets of faces are initially considered, the time needed for the selection of the best one clearly increases with the number of faces, on the other hand, once the contact faces were selected, the determination of the contact points is not time consuming. Future work includes the determination of *regulus* grasp over sets of faces that only allow this type of non-coplanar grasp, as well as an exhaustive comparison of the obtained grasps with the optimum ones according to different optimizations criteria.

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